AN ANALYSIS OF WEST POINT GRADUATES' ATTRITION RATES AND THEIR USE IN PREDICTION MODELS

by

Richard Vincent Oehrlein

United States
Naval Postgraduate School

THESIS

AN ANALYSIS OF WEST POINT GRADUATES' ATTRITION RATES
AND
THEIR USE IN PREDICTION MODELS

by

Richard Vincent Oehrlein

APR 1970

This document has been approved for public release and sale; its distribution is unlimited.

T 35543

An Analysis of West Point Graduates' Attrition Rates and Their Use in Prediction Models

by

Richard Vincent Oehrlein
Major, United States Army
B.S., United States Military Academy, 1964

Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN OPERATIONS RESEARCH

from the

NAVAL POSTGRADUATE SCHOOL
April 1970

Thesis 0546
c.1

ABSTRACT

Data on attrition rates of West Point graduates is collected and analyzed. The departure intervals during which attrition occurs are classified as wartime or peacetime for the various class groups. When such departure intervals for successive class groups are given the same classification, attrition rates remain stationary over short time periods. In situations when the departure intervals are given dissimilar classifications, ratios of peacetime-wartime attrition rates are calculated. An analysis of such attrition rates leads to the formulation of a model to estimate attrition. Use of the model and areas for further study are outlined.

LIBRARY
NAVAL POSTGRADUATE SCHOOL
MONTEREY, CALIF. 93940

TABLE OF CONTENTS

I. SUMMARY ---------- 5

A. INTRODUCTION ---------- 5

B. PURPOSE ---------- 6

C. RESULTS ---------- 7

II. DATA COLLECTION ---------- 8

A. SOURCE OF DATA ---------- 8

B. DATA COLLECTION PROCEDURES ---------- 8

1. Type of data collected ---------- 8

2. Classes selected ---------- 9

3. Data accumulation ---------- 10

III. DATA ANALYSIS ---------- 15

A. STATIONARITY ---------- 15

B. FLUCTUATING ATTRITION RATES ---------- 16

1. Peacetime-wartime distinction ---------- 16

2. Ratios of peacetime-wartime attrition rates -------- 20

IV. THE MODEL ---------- 24

A. BASIC NOTATION ---------- 24

B. MODEL FORMULATION ---------- 25

C. COMPARISONS BETWEEN CALCULATED AND ACTUAL ATTRITION ---- 27

V. CONCLUSIONS ---------- 30

A. IMPLEMENTATION ---------- 30

B. FURTHER STUDY ---------- 34

1. Attrition rate analysis ---------- 34

2. Other factors for consideration ---------- 34

LIST OF REFERENCES ----------

INITIAL DISTRIBUTION LIST ----------

FORM DD 1473 ----------

I. SUMMARY

A. INTRODUCTION

The United States Military Academy, West Point, New York, is an important source of officers for the United States Army. West Point graduates have made countless contributions to the nation and have provided the leadership to successfully counter aggression in times of war. From its initial class of two graduates in 1802, West Point has grown to present day classes that graduate approximately 800 officers each year.

Graduates of the United States Military Academy have been trained to pursue a military career, and are thought of as career officers. In the early part of the twentieth century, a high percentage of West Point graduates served the country as career officers. More recently, an alarmingly high percentage of its graduates are leaving the service early in their careers. Such departure from the service (including death) will be termed attrition.

Initial research indicates that West Point classes in the early 1900's exhibited approximately 30% attrition in the first 20 years of service, 20% attrition during 21-30 years of service, and, as such, 50% of the graduates remained in the service for more than 30 years. Classes in the 1920-1940 era maintained 30% attrition in the first 20 years, but the attrition rate increased to about 55% during the next ten yers of service. Thus, the percentage of officers remaining in service for more than 30 years decreased to about 15%.

After World War II, West Point classes were attaining 30% attrition as early as the first ten years of service, and 40% attrition in the

first fifteen years of service. In a few cases where 20 years service for post World War II classes could be examined, close to 50% of the graduates had departed the service. This is a significant increase in attrition when compared to the 30% attrition rate during the first twenty years of service for classes prior to World War II.

B. PURPOSE

The purpose of this study is to collect and analyze data on attrition rates of West Point officers, and to use such data in the development of a prediction model. Much past data concerning attrition rates of West Point graduates is examined, so that a thorough understanding of past performance is known. The analysis of such data leads to specific observations, and provides guidelines for predicting the number of West Point officers who leave the service.

Attrition rates for a class are calculated for successive five year intervals of service up to thirty years of service. Thus, attrition rates are obtained for 1-5,6-10,11-15,16-20,21-25, and 26-30 years of service. In addition, attrition rates for voluntary departure are separated from those of involuntary departure. A model is then formulated that assists in predicting the number of West Point graduates of a certain class group, who voluntarily or involuntarily, leave the service in one of the specified five-year intervals mentioned previously.

There are many factors that have a large effect on attrition rates of West Point officers. Some of these include the world situation (peace or war), economic outlook of the United States, governmental control on required military obligation, general attitude toward the military, and so on. Finding causes for attrition is not the purpose

of this study, but some such causes have been of assistance in model formulation. Specifically, attrition rates during wartime or peacetime are easily discernible, and relationships between such rates are developed.

C. RESULTS

Once attrition rates for the various classes have been calculated, an analysis of such rates leads to interesting observations. Certain attrition rates have remained relatively constant, others have changed dramatically at certain points in time, and wartime attrition rates differ markedly from those of peacetime. Compiling such information leads to a model for predicting the attrition of West Point classes. Recently, similar work on model formulation of retention of regular Marine Corps officers has been completed, and it was shown that retention is stationary over short time periods [Reference 1].

The number of graduates that leave the service is dependent on the number who graduate and the attrition rates during the specific departure interval. The departure intervals are classified either peacetime or wartime for various class groups, since attrition rates differ considerably in the two cases. When the departure intervals for two successive class groups are both of the same classification (both peacetime or both wartime), the attrition rates do not change much from class group to class group. Thus, the expected number of graduates who leave the service can be estimated using the number of graduates of that class group and the attrition rates of the previous class group. However, when the departure intervals for two successive class groups are of different classification, relationships found between peacetime and wartime attrition rates are used in the prediction model.

II. DATA COLLECTION

A. SOURCE OF DATA

Each year the West Point Alumni Foundation publishes a volume entitled Register of Graduates and Former Cadets of the United States Military Academy [Reference 2]. This publication gives a listing of West Point graduates by class. The latest edition of this publicatio (1969) was employed in obtaining data for this study.

In addition to listing West Point graduates by class, information is furnished as to the present status of each officer. Thus, it can b determined whether an officer is still on active duty or whether he i out of the service. If a graduate is out of the service, the date of and reason (death, resignation, retirement, etc.) for his departure is listed.

This information is obtained either from the individual officer or from other sources that are available to the Alumni Foundation. Thus, there is some error in the publication; however, data as to whether a graduate is still in or out of the service is accurate.

B. DATA COLLECTION PROCEDURES

1. Type of Data Collected

We record three distinct situations for each graduate. An officer voluntarily leaves the service, he involuntarily leaves the service, or he remains in the service for more than thirty years. Voluntary departures from service include resignation prior to 20 years of service or retirement during the interval 20-30 years of service. Involuntary departures include either death, medical discharge or dismissal from the service prior to 30-years service. Those graduates

who remained in the service for more than 30 years are no longer considered for either voluntary or involuntary departure.

Discrete time intervals of five years are used in denoting an officer's departure from service. Such five-year intervals include years of service 1-5,6-10,11-15,16-20,21-25, and 26-30. In addition, the five-year interval for the class considered has to be completed at the time data was taken (1969). For example, the class of 1962 is only examined for attrition rates in the first five years of service, since years 6-10 for that class have not yet transpired.

2. Classes selected

Originally, data was collected on classes that graduated in the early 1900's. Such historical data proved interesting. Classes often numbered less than 100 graduates, and frequently, had 50% of their graduates remain in service over thirty years. Such small classes and high retention rates are not realistic at this point in time, thus such data was not further analyzed.

Nevertheless, we wanted to examine data on attrition rates of graduates who had been in the service for a long period of time. Thus, classes as far back as the 1930's are examined, so that attrition rates for years of service 21-25 and 26-30 could be calculated. In addition, data on retention of graduates serving more than thirty years could be analyzed.

Classes of the 1940's, 1950's, and early 1960's are also examined. All possible attrition rates for the applicable five-year intervals are calculated and analyzed. As such, the classes in the early 1960's can only be investigated for the first five years of service, while those in the late 1950's are examined for years of service 1-5 and 6-10.

Data was not collected on all classes from 1930 to the early 1960's. For each successive five-year group, three classes were selected for data collection (see Table 1).

Table 1. Classes selected for data collection.

FIVE YEAR GROUP	CLASSES
1930-1934	1931, 1932, 1933
1935-1939	1936, 1937, 1938
1940-1944	1941, 1942, 1943
1945-1949	1947, 1948, 1949
1950-1954	1952, 1953, 1954
1955-1959	1957, 1958, 1959
1960-1964	1960, 1961, 1962

3. Data Accumulation

Each graduate of the above listed classes was checked in the Register of Graduates 1969 [Reference 2], and if he had left the service his departure was noted in the appropriate five-year interval 1-5, 6-10, 11-15, 16-20, 21-25, or 26-30. Such departures were noted as either voluntary or involuntary, so that two types of attrition occurred during each of the five-year intervals. A voluntary attrition rate will be denoted by VAR in the study, while an involuntary attrition rate will be abbreviated by IVAR. The remaining graduates who did not fall into any of the attrition categories were designated as having served more than 30 years.

The basic results of the data collection are summarized in Table 2 and 3. Table 2 contains data on classes from 1930-1944 (pre World War II) while Table 3 contains data on classes from 1945-1964 (post World War II). Tables 2 and 3 are similar in construction. The first column lists the departure intervals considered for the classes, while column two differentiates between voluntary (V) and involuntary (I) departure. Departure intervals up to twenty-years service are shown for post World War II classes, since such classes have not completed the years of service for the remaining departure intervals.

The remaining columns are headed by the specific classes being examined. Under each class the number of graduates that left the service, voluntarily or involuntarily, are listed for the appropriate departure interval. In addition, the percentage of officers of a particular class that each numbered entry represents is shown next to those numbers. The last line which totals the numbers and percentages for each class is only complete for the 1930's, since the other classes have not passed through all the departure intervals. In such cases, the number of officers of a class who have left the service are added, and this figure is listed in the numerator when a fraction appears in the "total row." The denominator designates the number of graduates of the class. The percentage figure for this fraction is also listed.

Table 2. Attrition of Pre World War II Classes.

Departure Intervals		1931		1932		1933		1936		1937		1938		1941		1942		1943	
		#	%	#	%	#	%	#	%	#	%	#	%	#	%	#	%	#	%
1-5	V	11	3.7	10	3.8	12	3.5	6	2.1	6	2.0	7	2.3	7	1.7	16	4.3	10	2.4
	I	9	3.0	12	4.6	20	5.8	10	3.6	15	5.0	16	5.3	61	14.4	84	22.4	82	20.0
6-10	V	6	2.0	5	1.9	7	2.0	4	1.5	6	2.0	6	2.0	16	3.8	7	1.9	8	2.0
	I	9	3.0	6	2.3	10	2.9	28	10.1	41	13.7	35	11.6	11	2.6	4	1.1	12	2.9
11-15	V	5	1.7	6	2.3	4	1.2	10	3.6	10	3.4	9	3.0	12	2.8	8	2.1	19	4.6
	I	30	10.1	29	11.1	39	11.3	9	3.3	6	2.0	6	2.0	10	2.4	6	1.6	11	2.7
16-20	V	8	2.7	4	1.5	7	2.0	10	3.6	13	4.4	11	3.7	39	9.2	20	5.3	29	7.1
	I	7	2.4	7	2.7	8	2.3	4	1.5	7	2.4	13	4.3	5	1.2	8	2.1	6	1.5
21-25	V	14	4.7	15	5.7	28	8.0	37	13.4	35	11.7	38	12.6	85	20.0	63	16.8	66	16.1
	I	4	1.4	6	2.3	7	2.0	4	1.5	8	2.7	3	1.0	9	2.1	4	1.1	5	1.2
26-30	V	123	41.4	105	40.0	129	37.2	107	38.8	109	36.6	104	34.6						
	I	24	8.1	20	7.7	31	8.9	13	4.7	12	4.0	16	5.3						
>30		47	15.8	37	14.1	45	12.9	34	12.3	30	10.1	37	12.3						
TOTAL		297	100.0	262	100.0	347	100.0	276	100.0	298	100.0	301	100.0	255/424	60.2	220/374	58.7	248/409	60.5

Table 3. Attrition of Post World War II Classes

Depart Intervals		1947		1948		1949		1952		1953		1954	
		#	%	#	%	#	%	#	%	#	%	#	%
1-5	V	33	10.6	33	11.0	84	14.6	113	21.4	113	22.1	122	19.3
	I	27	8.7	23	7.6	59	10.3	21	4.0	14	2.7	20	3.2
6-10	V	41	13.2	42	13.9	66	11.5	22	4.2	30	5.9	41	6.5
	I	5	1.6	5	1.7	8	1.4	8	1.5	9	1.8	12	1.9
11-15	V	17	5.5	10	3.3	15	2.6	11	2.1	22	4.3	31	4.9
	I	4	1.3	4	1.3	6	1.0	12	2.3	14	2.7	14	2.2
16-20	V	36	11.6	23	7.6	33	5.8						
	I	5	1.6	3	1.0	4	0.7						
Total		168/310	54.1	143/301	47.4	275/574	47.9	187/527	35.5	202/512	39.5	240/633	38.0

Depart Intervals		1957		1958		1959		1960		1961		1962	
		#	%	#	%	#	%	#	%	#	%	#	%
1-5	V	90	16.5	95	16.6	106	21.6	120	21.8	104	19.5	100	16.6
	I	12	2.2	15	2.6	20	4.0	23	4.2	22	4.1	32	5.3
6-10	V	37	6.8	42	7.3	29	5.8						
	I	13	2.4	14	2.4	13	2.6						
Total		152/546	27.9	166/573	28.9	168/499	34.0	143/550	26.0	126/534	23.6	132/601	21.9

From data in the previous two tables, we calculate average attrition rates for the three year class groups. These are shown in Table 4. This table is similar to the previous two tables, except the columns are headed by class groups and not classes. The number of graduates that leave the service for the classes within a specific three year group are added so that one figure is obtained for a VAR and one for an IVAR during the designated departure interval. The percentages that such numbers represent for the three-year class group are also listed. When a three year group has not completed the years of service for specific departure intervals, fractions appearing in the "total row" are interpreted as in the previous two tables.

Table 4. Attrition of Three Year Class Groups .

Departure Interval		1931-1933		1936-1938		1941-1943		1947-1949		1952-1954		1957-1959		1960-196	
		#	%	#	%	#	%	#	%	#	%	#	%	#	%
1-5	V	33	3.6	19	2.2	33	2.7	150	12.7	348	20.8	291	18.0	324	19.
	I	41	4.5	41	4.7	227	18.8	109	9.2	55	3.3	47	2.9	77	4.
6-10	V	18	2.0	16	1.8	31	2.6	149	12.6	93	5.6	108	6.7		
	I	25	2.8	104	11.9	27	2.2	18	1.5	29	1.7	40	2.5		
11-15	V	15	1.7	29	3.3	39	3.2	42	3.5	64	3.8				
	I	98	10.8	21	2.4	27	2.2	14	1.2	40	2.5				
16-20	V	19	2.1	34	3.9	88	7.3	92	7.8						
	I	22	2.4	24	2.7	19	1.6	12	1.0						
21-25	V	57	6.3	110	12.6	214	17.7								
	I	17	1.9	15	1.7	18	1.5								
26-30	V	357	39.4	320	36.6										
	I	75	8.3	41	4.7										
>30		129	14.2	101	11.5										
						720		586		629		486		401	
TOTAL		906	100.0	875	100.0	1207	59.8	1185	49.5	1672	37.7	1618	30.1	1685	23.

III. DATA ANALYSIS

A. STATIONARITY

A stationary distribution is one which remains unchanged over time, so the probabilities of being in specified states remain constant independent of the time period considered [Ref. 3]. Certain attrition rates of West Point graduates can be seen to be stationary for short time periods. Fairly stable attrition rates, both voluntary and involuntary, for specified departure intervals are exhibited by the classes within a three year group. Certain data extracted from Tables 2 and 3 are shown in Table 5 to illustrate such stationarity. The entries next to each class represent the attrition rates for the departure interval specified at the head of the column. The V or I next to the five year time interval refers to voluntary or involuntary respectively.

Table 5. Examples of Stationary Attrition Rates

CLASS	TIME INTERVAL		CLASS	TIME INTERVAL	
	(1-5)V	(11-15)I		(6-10)V	(16-20)I
1931	3.7	10.1	1947	13.2	1.6
1932	3.8	11.3	1948	13.9	1.0
1933	3.5	11.3	1949	11.5	0.7
	(6-10)I	(21-25)V		(1-5)V	(6-10)I
1936	10.1	13.4	1957	16.5	2.4
1937	13.7	11.7	1958	16.6	2.4
1938	11.6	12.6	1959	21.6	2.6

Further reference to Tables 2 and 3 can be made to further illustrate that attrition rates are stable for the three classes within the three year group.

In addition to stationarity between classes within a three year group, average attrition rates for a group frequently did not differ much from those of the next group. Generally, this occurred when the world situation (peace or war) remained the same. For example, class group 1936-1938 had a VAR of 1.8% during years of service (6-10) while that of class group 1931-1933 was 2.0% for (6-10) years of service. Also, the VAR for class group (1960-1962) was 19.2% during (1-5) years of service, while that of class group (1957-1959) was 18.0% for (1-5) years of service. Other examples can be cited from Table 4, and will be discussed in more detail in a later section.

B. FLUCTUATING ATTRITION RATES

1. Peacetime/Wartime Distinction

We have just noted that attrition rates for specific time intervals experienced by a three year class group were often similar to those experienced by the previous three year group. Such was the case when the world situation (peace or war) remained the same. However, if one three year class group confronted a wartime environment for a specific interval, and the following three year group encountered a peacetime environment for that same interval (years of service), different attrition rates resulted. For example, the three year class group 1947-1949 felt the brunt of the Korean War during its first five years of service, while class group 1952-1954 experienced a peaceful environment during its initial five year service. Such situations are identified for all the classes examined, and relationships are then developed between peacetime and wartime attrition rates.

To examine such relationships in depth, attrition rates for the specified years of service have to be classified as either peacetime

or wartime. For example, a three year class group could encounter peacetime during years of service (1-5), wartime during years of service (6-10), and so on. Intuitively, one expects VAR's to be lower during wartime periods, since governmental controls can be imposed which curb the departure of officers from service. Also, it should be intuitive that IVAR's will be higher during wartime periods, since more deaths and medical discharges will result.

The classification of attrition rates as either peacetime or wartime is summarized in Table 6. The three year class groups are listed, and the voluntary (V) attrition rates are again separated from those that are involuntary (I). The remaining columns are headed by departure intervals for which attrition rates are classified. Beyond twenty years of service it appears that the war or peace classification does not noticeably affect attrition rates. Instead of actual attrition rates, the letters P or W are used to indicate whether the attrition for a class group during the specified interval is classified as peacetime or wartime.

Table 6. Classification of Attrition Rates

CLASS GROUP	V or I	1-5	6-10	11-15	16-20
1931-1933	V	P	W	W	W
	I	P	P	W	W
1936-1938	V	W	W	P	W
	I	P	W	P	W
1941-1943	V	W	W	P	P
	I	W	W	P	P
1947-1949	V	W	P	P	P
	I	W	P	P	P
1952-1954	V	P	P	P	
	I	P	P	W	
1957-1959	V	P	P		
	I	P	W		
1960-1962	V	P			
	I	W			

In most cases, the classification of attrition rates as either peacetime or wartime rates is straight forward. However, some class groups have a VAR classified as peacetime, and an IVAR as wartime or vice versa for the same time interval. Such apparent discrepancies are discussed below.

Prior to discussing such discrepancies, certain observations will be made. Any time the United States was subjected to a wartime environment, the VARs were generally classified wartime, since governmental control made it difficult to leave the service. If governmental control was not imposed during periods of conflict, the VARs were classified peacetime.

In the classification of IVARs, certain factors will be discussed. First, a war could have certain phases during which it had no noticeable effect on the death rates. This occurs when the United States plays a non-combatant role for specific wartime periods, and the IVARs for such periods are classified peacetime. Second, death rates of higher ranked officers often remain low during wartime. In such cases the IVARs are still classified wartime, since deaths are attributed to war. The observations just made will be further illustrated below.

Class group 1931-1933 has a VAR designature as wartime for 6-10 years of service, while its IVAR for that time interval is designated peacetime. Six to ten years of service were the years 1937-1941, 1938-1942, and 1939-1943 for the classes 1931, 1932, and 1933 respectively. As such, the impact of World War II made it difficult for officers to resign from the service during this time interval, and thus the VAR is designated wartime. However, most of the deaths in World War II occurred in the heavy fighting in 1944 and 1945; thus (6-10) years of

service for the group 1931-1933 did not include 1944 and 1945. Hence, the IVAR is designated peacetime, exemplifying the noncombatant concept mentioned previously.

Class group 1931-1933 also has a wartime VAR and a wartime IVAR for 16-20 years of service. This time period coincided with the Korean War. Here the resignation rate was low due to governmental controls, so the VAR is classified wartime. The IVAR is also classified wartime, but the death rates of such higher ranked officers was comparatively low.

The next three year class group, 1936-1938, is given a wartime VAR and a peacetime IVAR during (1-5) years of service. Since the first five years of service were the years 1936-1941, 1937-1942, and 1938-1943 for classes 1936, 1937, and 1938 respectively, the same reasoning discussed above for class group 1931-1933, for its 6-10 years of service, applied here. Also, class group 1936-1938 was similarly affected by the Korean War during its 16-20 years of service as the previous class group 1931-1933. Thus, the same classification of wartime VAR and wartime IVAR is made.

Year Group 1941-1943 encountered World War II immediately upon graduation, and both attrition rates for the first five years of service are given a wartime designation. Then the Korean War which occurred in the neighborhood of 8 years service for this class group prompted the classification of wartime attrition rates for 6-10 years of service. Again the death rate of higher ranked officers was quite low.

The post World War II classes have been affected by both the Korean War and Vietnam War. 1947-1949 is the first post World War II

class group, and both the VAR and IVAR during the first five years of service are designated wartime rates due to the Korean War.

The remaining class groups 1952-1954, 1957-1959, and 1960-1962 were all somewhat affected by the Vietnam War. However, certain observations made previously when discussing "conventional type wars" (World War II and the Korean War) were no longer true when analyzing the "guerrilla war" in Vietnam. First, governmental controls had not been imposed during the departure intervals considered for the study, so the VARs are classified peacetime. Eventually, the government did retain officers beyond their expiration dates but such data was not available at the time of the study. Second, death rates of higher ranked officers were noticeably affected due to the nature of a guerrilla war, and the classification of the IVARs as wartime is stronger than that made previously.

2. Ratios of Peacetime-Wartime Attrition Rates

Now that the attrition rates for the class groups have been classified as peacetime or wartime, ratios between such dissimilar rates are developed. This occurred when an attrition rate for one class group during a certain departure interval is designated peacetime, and the attrition rate for the next group during that same departure interval is a wartime rate or vice versa. Since pre World War II classes are divided into three class groups, three VARs and three IVARs could be listed for each departure interval. Thus, when two VARs are designated peacetime and the remaining attrition rate for the same departure interval is a wartime rate, ratios are developed.

In making calculations in developing such ratios, a basic "averaging" procedure was implemented. First, an overall rate was

calculated for the two attrition rates that are classified the same for a departure interval (e.g., peacetime). This involved summing the number of graduates who had left the service during that time interval from both class groups, and then that sum was divided by the total number of graduates of both groups. As a result, one attrition rate for peacetime is obtained for a specific departure interval, and this could then be related to the third attrition rate for that interval, which would be a wartime rate.

The same procedure held for development of ratios between peacetime and wartime rates for post World War II classes; however, four class groups are now considered. Each five year departure interval could have four VARs and four IVARs. For certain departure intervals, the more recent class groups do not have attrition rates, since the class group has not completed so much time in the service.

In the departure interval 1-5 for post World War II classes, two class groups are given a peacetime IVAR and the other two class groups have a wartime IVAR. But one wartime rate pertains to the Korean War and the other one pertains to the Vietnam War. The two peacetime rates were averaged as discussed previously and a ratio was calculated between this rate and each one of the wartime rates.

The ratios between the peacetime and wartime rates are tabulated below in Table 7. The data was extracted from Tables 4 and 6. Column one lists the time intervals for departure, and column two differentiates between voluntary and involuntary. Column three lists the peacetime attrition rates, and column four the wartime attrition rates. Column five shows the ratios of such attrition rates, where subscripts p and w refer to peacetime and wartime respectively. Below the table is a sample calculation of a tabulated result.

Table 7. Ratios of Peacetime Wartime Attrition Rates.
Pre World War II Classes.

Departure Interval		Att. Rate (Peace %)	Att. Rate (War %)	Ratios
1-5	V	3.6	2.5	$VAR_w = 0.70\ VAR_p$
	I	4.6	18.8	$IVAR_w = 4.08\ IVAR_p$
6-10	I	2.8	11.9	$IVAR_w = 4.25\ IVAR_p$*
11-15	V	3.3	1.7	$VAR_w = 0.52\ VAR_p$
	I	2.3	10.8	$IVAR_w = 4.70\ IVAR_p$
16-20	V	7.3	3.0	$VAR_w = 0.41\ VAR_p$
	I	1.6	2.6	$IVAR_w = 1.62\ IVAR_p$

Post World War II Classes.

1-5	V	19.3	12.7	$VAR_w = 0.66\ VAR_p$
	I	3.1	9.2 Korean	$IVAR_w = 2.97\ IVAR_p$
			4.6 Vietnam	$IVAR_w = 1.48\ IVAR_p$
6-10	I	1.6	2.5	$IVAR_w = 1.56\ IVAR_p$
11-15	I	1.2	2.5	$IVAR_w = 2.08\ IVAR_p$

Note: *IVAR for class group 1941-1943 not considered.
(low death rate of higher ranked officers)

Sample Calculation:

Pre World War II classes: departure interval (1-5)V reference Tables 4 and 6.

Class Group	# in class gp.	# that departed	Classif.
1931-1933	906	33	P
1936-1938	875	19	W
1941-1943	1207	33	W

$$VAR_w = \frac{19 + 33}{875 + 1207} = 2.5\%$$

$$VAR_p = \frac{33}{906} = 3.6\%$$

Table 7 indicates that VARs during wartime were approximately 0.4 -- 0.7 times the VARs during peacetime for the specific departure intervals listed. There was more variation in the ratios of IVARs. IVARs during World War II were approximately 4-5 times the IVARs during peacetime, while the IVARs for the Vietnam War were 1.5 -- 2.1 times the peacetime IVARs.

IV. THE MODEL

A. BASIC NOTATION

To develop a model, mathematical representation of concepts discussed previously must be introduced. Basic notation for class groups, attrition rates, departure intervals, etc. will be introduced. Such notation will serve as a guide for the model discussion.

Previously, the number of graduates for each three year group were listed in Table 4. Those numbers represent the starting state of the system and are designated N_o. The three year class groups are denoted by the interval (a-b), where (a-b) includes (1931-1933), (1936-1938), (1941-1943), (1947-1949), (1952-1954), (1957-1959) and (1960-1962). Thus N_o (a-b) represents the number of graduates of a specific three year group (a-b).

Vars and IVARs of both peacetime and wartime classification are represented as follows:

$P_{ij}(a\text{-}b;x)$ = the fraction of graduates from class group (a-b) who leave for reason i in period j given that period j has x classification .

In this notation i=1 refers to a VAR and i=2 to an IVAR. The range of j is 1,2---,6, where such numbers denote departure intervals 1-5, 6-10,---,26-30 respectively. Also x=p refers to a peacetime classification for interval j and x=w to a wartime classification.

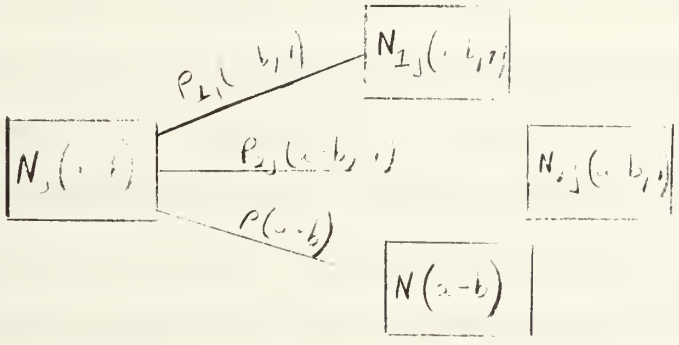
In addition to the two types of attrition, the retention of officers beyond 30 years service was another state investigated. So the retention rate is designated P(a-b) for a specified class group. P(a-b) is just 1 minus the faction of officers who left the service prior to 30 years. Thus $P(a\text{-}b) = 1 - \sum_{i=1}^{2} \sum_{j=1}^{6} P_{ij}(a\text{-}b;x)$.

The purpose of the model is to predict attrition of graduates from a class group. The number from class group (a-b) that leave the service for reason i in the departure interval j where j is of classification x is denoted $N_{ij}(a\text{-}b;x)$. The number of graduates that serve more than 30 years is indicated by $N(a\text{-}b)$.

Two more notational aids will be introduced prior to discussing the model. Whenever reference is made to the most recent class group prior to the one being analyzed, the notation $(a\text{-}b;x)^*$ is used. Also, the peacetime-wartime ratios listed in Table 7 are represented by $R(x,y)$. Such ratios are required when departure interval j for two successive class groups is of different classification (peace versus war).

B. MODEL FORMULATION

The basic inputs to the model are the number of graduates of the class group and the attrition rates for the specific departure intervals. Using the notation just described, the system is basically the following:



The boxes represent the number of graduates in the various states discussed previously while $P_{1j}(a\text{-}b;x)$ and $P_{2j}(a\text{-}b;x)$ denote attrition rates, and $P(a\text{-}b)$ is the retention rate.

It has been demonstrated that attrition rates for one class group are often similar to those of the previous class group. This is important in that attrition rates during departure intervals for a class group under study may not be known. Applying the stationarity

principle, attrition rates for such departure intervals of the previous class group can be used in estimating the number of graduates that leave the service of the class group being studied.

Basic equations established to estimate the number of graduates of a class group that depart the service include the following:

$$4.1 \qquad N_{ij}(a\text{-}b;x) = N_o(a\text{-}b)\ P_{ij}(a\text{-}b;y)^*\ R(x,y).$$

The letters x and y refer to peacetime and wartime or vice versa, so the departure interval specified by j is of dissimilar classification for the two class groups. R(x,y) pertains to the peacetime-wartime ratios listed in Table 7. The class group being studied is (a-b) while (a-b)* refers to the previous class group. So the number of graduates of class group (a-b) is multiplied by the proper attrition rate of the previous class group, and the appropriate peacetime-wartime ratio in order to estimate the number of graduates departing the service for the specified i and j.

$$4.2 \qquad N_{ij}(a\text{-}b;x) = N_o(a\text{-}b)\ P_{ij}(a\text{-}b;x)^*.$$

In this equation the departure interval specified by j is given the same classification for both class groups (both peace or both war). Thus, $R(x,y) = R(y,x) = 1$. Such situations illustrate the stationarity concept between class groups; hence, attrition rates of the previous class group are multiplied by the number of graduates of the class group under study to obtain an estimate of the number departing the service for specified i and j.

$$4.3 \qquad N(a\text{-}b) = N_o(a\text{-}b)\ P(a\text{-}b)^*.$$

This equation gives the number serving more than 30 years. The retention rate of the previous class group is multiplied by the number in the starting state of the class group being examined to estimate the number

of graduates serving more than 30 years. Also, $N(a\text{-}b) = N_o(a\text{-}b) - \sum_{i=1}^{2} \sum_{j=1}^{6} N_{ij}(a\text{-}b;x)$ since the right hand side of the equation is the difference between the number of graduates and the number who left the service prior to 30 years.

$$4.4 \qquad N_{1j}(a\text{-}b;x) = \sum_{j=1}^{6} N_{1j}(a\text{-}b;x),$$

$$4.5 \qquad N_{2j}(a\text{-}b;x) = \sum_{j=1}^{6} N_{2j}(a\text{-}b;x).$$

Expressions 4.4 and 4.5 represent total voluntary attrition and total involuntary attrition respectively. An expression for total attrition prior to 30 years service is $N_{1j}(a\text{-}b;x) + N_{2j}(a\text{-}b;x)$.

C. COMPARISONS BETWEEN CALCULATED AND ACTUAL ATTRITION

In this study actual data was collected on the attrition of graduates. Using equations 4.1 or 4.2 the predicted attrition for class group (a-b) is found. Such predicted attrition is compared with the known actual attrition.

Table 8 shows some results of the predicted versus actual attrition. Column one lists the class group and column two specifies the number of graduates of that class group. Column three designates the applicable attrition rate of the last class group required to estimate the attrition for a certain interval of the class group under study. Column four denotes the applicable wartime-peacetime ratios, and column five contains the calculated attrition. The actual attrition is then shown in column six. In most cases, the predicted attrition is very close to the actual attrition.

Table 8. Predicted Attrition versus Actual Attrition.

Class Gp.	$N_o(a\text{-}b)$	$P_{ij}(a\text{-}b;x)$*	$R(x,y)$	$N_{ij}(a\text{-}b;x)$	Actual Att.
1936-1938	875	$P_{11}(31\text{-}33;_p)=.036$	$VAR_w=.70VAR_p$	$N_{11}(36\text{-}38;_w)=22$	19
		$P_{12}(31\text{-}33;_w)=.020$	1	$N_{12}(36\text{-}38;_w)=18$	16
		$P_{13}(31\text{-}33;_w)=.017$	$VAR_w=.52VAR_p$	$N_{13}(36\text{-}38;_p)=29$	29
		$P_{21}(31\text{-}33;_p)=.045$	1	$N_{21}(36\text{-}38;_p)=39$	41
		$P_{23}(31\text{-}33;_w)=.108$	$IVAR_w=4.70IVAR_p$	$N_{23}(36\text{-}38;_p)=20$	21
		$P_{24}(31\text{-}33;_w)=.024$	1	$N_{24}(36\text{-}38;_w)=21$	24
1941-1943	1207	$P_{11}(36\text{-}38;_w)=.022$	1	$N_{11}(41\text{-}43;_w)=27$	33
		$P_{12}(36\text{-}38;_w)=.018$	1	$N_{12}(41\text{-}43;_w)=22$	31
		$P_{13}(36\text{-}38;_p)=.033$	1	$N_{13}(41\text{-}43;_p)=40$	39
		$P_{21}(36\text{-}38;_p)=.047$	$IVAR_w=4.08IVAR_p$	$N_{21}(41\text{-}43;_w)=232$	227
		$P_{25}(36\text{-}38;_p)=.017$	1	$N_{25}(41\text{-}43;_p)=20$	18
1947-1949	1185	$P_{13}(41\text{-}43;_p)=.032$	1	$N_{13}(47\text{-}49;_p)=38$	42
		$P_{14}(41\text{-}43;_p)=.073$	1	$N_{14}(47\text{-}49;_p)=87$	92
1952-1954	1672	$P_{11}(47\text{-}49;_w)=.127$	$VAR_w=0.66VAR_p$	$N_{11}(52\text{-}54;_p)=322$	348
		$P_{13}(47\text{-}49;_p)=.035$	1	$N_{13}(52\text{-}54;_p)=59$	64
		$P_{21}(47\text{-}49;_w)=.092$	$IVAR_w=2.97IVAR_p$	$N_{21}(52\text{-}54;_p)=52$	55
		$P_{22}(47\text{-}49;_p)=.015$	1	$N_{22}(52\text{-}54;_p)=25$	29
1957-1959	1618	$P_{12}(52\text{-}54;_p)=.056$	1	$N_{12}(57\text{-}59;_p)=91$	108
		$P_{21}(52\text{-}54;_p)=.033$	1	$N_{21}(57\text{-}59;_p)=53$	47
		$P_{22}(52\text{-}54;_p)=.017$	$IVAR_w=1.56IVAR_p$	$N_{22}(57\text{-}59;_w)=43$	40
1960-1962	1685	$P_{12}(57\text{-}59;_p)=.18$	1		

There were certain instances when attrition did not remain stationary during departure intervals of similar classification for successive class groups. This was most evident for attrition during the departure interval 21-25. Referring to Table 4 indicates that voluntary attrition was increasing during that interval for successive class groups. But there was a small decrease in attrition during the 26-30 interval and in retention beyond 30 years for those class groups. This indicates that graduates were retiring from the service at an earlier period of their careers.

V. CONCLUSIONS

The conclusion reached from this study is that attrition rates of graduates are fairly stable over short time periods when the world situation remains the same. Such stationarity permits us to use data on past attrition rates to estimate the attrition of the class group being studied. When the world situation changes (peace to war or vice versa) attrition during the same departure interval for successive class groups varies. Ratios between wartime and peacetime rates were then calculated so that a better understanding resulted in such variation of attrition rates.

A. IMPLEMENTATION

To implement the work done in this study one fact must be kept in mind. To predict future attrition for a specific departure interval, the last previous class group with actual attrition rates for that same departure interval is used. So, this model only looks five years into the future. For example, data on attrition for the first five years of service for a present class group 1965-1967 could be based on data obtained from class group 1960-1962 for its first five year departure interval. However, data on six to ten years of service for class group 1965-1967 would be based on attrition of class group 1960-1962 for its 6-10 years of service. This is not available yet, in that 6-10 years of service for class group 1960-1962 will not be completed till 1972.

Past class groups can also be further examined for attrition. Attrition on such class groups is known up till a specific departure

interval, but then attrition for the next five year departure interval could be estimated using the data of the previous three year class group that just completed its service for that time period. For example, class group 1952-1954 has data on attrition for the first fifteen years of service, but attrition during 16-20 years of service could be estimated using the known attrition that class group 1947-1949 exhibited during its 16-20 years of service.

Certain situations will be discussed to show how the work in this study can be implemented. Either the prediction of attrition for present class groups or that of past class groups will be considered. Table 9 summarizes the results of such predictions. Columns one, two, and three list the class groups, number of graduates for the groups, and type of attrition (V or I). Column four lists the departure interval for which attrition is predicted, and the w or p classifies the interval as wartime or peacetime. Since such departure intervals have not been completed for the class groups being studied, both classifications are considered. Then the estimated attrition is shown in column five. Below the table are sample calculations.

One further result from the study is a table on the cumulative attrition of class groups. The seven class groups analyzed are listed across the top, and the cumulative years of service are denoted along the left hand side. The attrition rates (in percentages) listed are cumulative totals up to and including the year of service in the left hand column. Under each cumulative attrition rate is a cumulative total on VARs listed first and a cumulative total on IVARs. The cumulative attrition rates are shown in Table 10.

Table 9. Prediction of Attrition for Years 1966-1974 .

Class Group	N_o(a-b)	V or I	Departure Interval	Prediction
1966-1968	1868	V	$(1\text{-}5;_w)$	239
		V	$(1\text{-}5;_p)$	358
		I	$(1\text{-}5;_w)$	86
		I	$(1\text{-}5;_p)$	57
1960-1962	1685	V	$(6\text{-}10;_p)$	113
		I	$(6\text{-}10;_w)$	42
		I	$(6\text{-}10;_p)$	27
1957-1959	1618	V	$(11\text{-}15;_w)$	32
		V	$(11\text{-}15;_p)$	61
		I	$(11\text{-}15;_w)$	40
		I	$(11\text{-}15;_p)$	19
1952-1954	1672	V	$(16\text{-}20;_w)$	54
		V	$(16\text{-}20;_p)$	131
		I	$(16\text{-}20;_p)$	17
		I	$(16\text{-}20;_w)$	27

Sample Calculations:

1966-1968: $N_{11}(1966\text{-}1968;_w) = N_o('66\text{-}68)\ P_{11}('60\text{-}'62;_p)R_{(x,y)}$

$= (1868)\ (.192)\ (0.66) = \underline{\underline{239}}$

$N_{11}(1966\text{-}1968;_p) = N_o('66\text{-}'68)\ P_{11}('60\text{-}'62;_p)$

$= 1868\ (.192) = \underline{\underline{358}}$

Table 10. Cumulative Attrition.

	1931-33	1936-38	1941-43	1947-49	1952-54	1957-59	1960-62
5 yrs	8.1 3.6,4.5	6.9 2.2, 4.7	21.5 2.7, 18.8	21.9 12.7, 9.2	24.1 20.8, 3.3	20.9 18.0, 2.9	23.8 19.2,4.6
10yrs	12.9 5.6, 7.3	20.6 4.0, 16.6	26.3 5.3, 21.0	36.0 25.3, 10.7	31.4 26.4, 5.0	30.1 24.7, 5.4	
15yrs	25.4 7.3, 18.1	26.3 7.3, 19.0	31.7 8.5, 23.2	40.7 28.8, 11.9	37.7 30.2, 7.5		
20yrs	29.9 9.4, 20.5	32.9 11. 2, 21.7	40.6 15.8, 24.8	49.5 36.6, 12.9			
25yrs	38.1 15.7, 22.4	47.2 23.8, 23.4	59.8 33.5, 26.3				
30yrs	85.8 55.1, 30.7	88.5 60.4, 28.1					
>30	100.0	100.0					

B. FURTHER DEVELOPMENT

1. Attrition Rate Analysis

For this study, attrition rates for three year class groups were calculated, analyzed and used in the prediction model. Other methods of calculating and utilizing attrition rates could be investigated. For example, attrition data on five consecutive classes for specific time periods could be obtained. Then attrition for the next class during the designated time period could be estimated using average attrition for the past five classes. Further study could examine such various procedures of obtaining good estimates of attrition.

Distribution theory for attrition rates is an area that could be studied. Attrition rates could be assumed to have a certain distribution, and actual data could be applied to test the goodness of fit of such a distribution. The assumption of loss rate being constant independent of time spent in the system, sometimes made by Bartholomew [Ref. 4] could not apply here. Exponential distributions that assume constant attrition, independent of outside forces, would not be realistic for this study.

Such completed length of service distributions [Ref. 4] are an alternative way of expressing loss intensities. If distributions could approximate such attrition, the mean and variance of time in service of a West Point graduate could be readily obtained.

2. Other Factors for Consideration

Since attrition is dependent on many other factors other than the peacetime-wartime classification of intervals used in the study, such factors could be considered. For example, the prosperity of the

United States economy has briefly been alluded to as being quite influential on attrition rates. As such, there were very low resignation rates during the depression, but the present day resignation rates are getting very high. The lure of good civilian jobs is drawing officers away from the service. Such an important outside force could be incorporated into the model.

Such a study on attrition rates has indicated that a high percentage of graduates are now leaving the service early in their careers. Thus, the methods utilized in the selection of personnel for enrollment to the academy, and the required service obligation upon graduation are two areas that should be constantly analyzed. A West Point education is very costly to the government and with the increased emphasis on cost reduction in the government, cost effectiveness studies on the various sources of officer commissions may be applicable.

The methodology utilized in this study could be applied to a study on attrition of the U. S. Army. The number of soldiers that constitute the starting state at some point in time could be established. Past attrition rates of both enlisted personnel and officers could be analyzed. Relationships involving the above two factors and other pertinent factors to the system could be incorporated into a model that would assist in predicting attrition for the entire United States Army.

LIST OF REFERENCES

1. McAfee, Carlos K, A Cohort Model for Predicting Retention of Regular Marine Corps Officers, Masters Thesis, Naval Postgraduate School, 1970.

2. Register of Graduates and Former Cadets of the United States Military Academy, West Point Alumni Foundation, Inc., 1969.

3. Feller, William, An Introduction to Probability Theory and its Applications, 2d ed., v. 1, John Wiley & Sons, 1966.

4. Bartholomew, David J., Stochastic Models for Social Processes, John Wiley & Sons, 1967.

INITIAL DISTRIBUTION LIST

		No. Copies
1.	Defense Documentation Center Cameron Station Alexandria, Virginia 22314	20
2.	Library, Code 0212 Naval Postgraduate School Monterey, California 93940	2
3.	Department of Operations Analysis, Code 55 Naval Postgraduate School Monterey, California 93940	1
4.	Asst. Professor K. T. Marshall, Code 55mt Department of Operations Analysis Naval Postgraduate School Monterey, California 93940	5
5.	Major Richard V. Oehrlein 340 Delaware Avenue Union, New Jersey 07083	2
6.	Mr. John Kraft Deputy Director of Research Admissions Office United States Military Academy West Point, New York 10096	1
7.	Civil Schools Branch Office of Personnel Operations Washington, D. C. 20315	1

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

GINATING ACTIVITY *(Corporate author)* val Postgraduate School nterey, California 93940	2a. REPORT SECURITY CLASSIFICATION Unclassified 2b. GROUP

PORT TITLE

Analysis of West Point Graduates' Attrition Rates and Their Use in ediction Models

SCRIPTIVE NOTES *(Type of report and, inclusive dates)*

ster's Thesis; (April 1970)

THOR(S) *(First name, middle initial, last name)*

chard Vincent Oehrlein

PORT DATE ril 1970	7a. TOTAL NO. OF PAGES 40	7b. NO. OF REFS 4
ONTRACT OR GRANT NO. ROJECT NO	9a. ORIGINATOR'S REPORT NUMBER(S)	
	9b. OTHER REPORT NO(S) *(Any other numbers that may be assigned this report)*	

STRIBUTION STATEMENT

is document has been approved for public release and sale; its distribution unlimited.

UPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY Naval Postgraduate School Monterey, California 93940

BSTRACT

Data on attrition rates of West Point graduates is collected and analyzed. e departure intervals during which attrition occurs are classified as wartime peacetime for the various class groups. When such departure intervals for ccessive class groups are given the same classification, attrition rates remain ationary over short time periods. In situations when the departure intervals e given dissimilar classifications, ratios of peacetime-wartime attrition rates e calculated. An analysis of such attrition rates leads to the formulation of a del to estimate attrition. Use of the model and areas for further study are tlined.

14 KEY WORDS	LINK A		LINK B		LINK	
	ROLE	WT	ROLE	WT	ROLE	
Personnel						
Prediction						
Attrition Rates for Personnel						

Thesis
0246
c.1

125672

Oehrlein
An analysis of West Point graduates' attrition rates and their use in prediction models.

12 DEC 72 22079
27 OCT 78 25816
7 DEC 79 26329
20 JUN 80 25669
[illegible] 26661
7 JUN 83 27859
14 OCT 83 27859
30 SEP 91 [illegible]

125672

Thesis
0246
c.1

Oehrlein
An analysis of West Point graduates' attrition rates and their use in prediction models.

thesO246
An analysis of West Point graduates' att
3 2768 000 99927 0
DUDLEY KNOX LIBRARY